# Truck-UAV Collaborative Emergency Management Under Multiple Uncertainties

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*Keywords:* humanitarian logistics, location-routing, truck–UAV collaboration, robust optimization

## 1 Introduction

The increasing frequency of natural disasters highlights the critical need for effective emergency management systems. Although technological advancements have improved the accuracy of predicting events such as typhoons and floods, complete knowledge is unapproachable due to the dynamic nature of disasters. The uncertainties on humanitarian relief demands and transportation infrastructure damage pose challenges for optimal pre-disaster preparedness and post-disaster response.

Emergency management involves a sequential decision-making process for planning, managing, and controlling the resource flows from supply to demand nodes (Sheu et al., 2007). Two major challenges in this process are the deployment of emergency relief resources and the routing of vehicles for relief delivery. Facility location models, such as maximal covering location models (Balcik and Beamon, 2008), have been commonly used for location problems. In contrast, vehicle routing problems (VRPs), known as NP-hard problems, have been adapted to include complexities such as multiple commodity types (Shen et al., 2009), split deliveries (Vitoriano et al., 2011), and demand clustering (Özdamar and Demir, 2012). Integrated location-routing models provide a more effective strategy. For instance, Dalal and Üster (2018) developed a stochastic programming framework incorporating uncertainties in disaster location and intensity, and later extended their model to include disaster duration and evacuee compliance (Dalal and Üster, 2021). These studies lay a strong foundation for comprehensive emergency management that jointly optimizes relief location and vehicle assignments for truck-only systems.

However, few studies have investigated the potential collaboration between trucks and unmanned aerial vehicles (UAVs). Post-disaster scenarios often involve damaged road infrastructure, which can severely restrict trucking. UAVs offer a complementary solution, as they are less dependent on roads, offering unique access to isolated areas. By combining the flexibility of UAVs with the larger cargo capacity of trucks, hybrid strategies have the potential to significantly improve the efficiency of disaster response.

Despite these advantages, the complexities in developing truck-UAV collaborative response systems are not yet adequately addressed. The integration of UAVs introduces several unique constraints: (i) UAV loading limitations, necessitating balanced relief distribution between trucks and UAVs; (ii) UAV range limitations, restricting the feasible launch and landing locations for UAV operations; and (iii) UAV retrieval, requiring synchronized assignments of trucks and UAVs. These constraints, combined with the truck routing problem, create significant challenges in developing effective solution methods. Consequently, existing approaches mainly rely on heuristics to address these complexities. For instance, Long et al. (2023) proposed a dynamic truck-UAV collaboration strategy for resilient emergency response using a tabu search-based integrated scheduling algorithm.

Moreover, existing literature on truck-UAV collaborative response often overlooks the dynamic nature of disasters. Disasters bring about evolving demand and unpredictable network disruptions. Wang et al. (2024) developed a hybrid heuristic algorithm to solve the truck-drone routing problem with stochastic demand. Unlike demand uncertainty, network disruptions and uncertain travel time significantly impact response efficiency, as potential delays can cascade, impacting subsequent vehicle movements and UAV consolidation. This poses significant challenges for coordinated routing and scheduling, which requires precise spatiotemporal truck-UAV synchronizations. The inclusion of location decisions further complicates this, introducing interdependence between assignment and location strategies. Therefore, specialized models and optimization algorithms are crucial, especially for large-scale networks with multiple uncertainties.

To address these challenges, we investigate a truck–UAV collaborative emergency management problem (TUEMP), which incorporates location decisions and synchronized vehicle assignments under travel time and demand uncertainties. We formulate a robust optimization model to tackle the uncertainties and develop tailored solution algorithms based on Benders decomposition. Our contributions are summarized as follows:

- We develop a robust optimization model for the TUEMP. This model is adaptable to varying levels of predictive information and coordinates UAV takeoffs and landings from trucks, ensuring synchronized delivery to meet relief demands.
- Through tractable reformulation, we develop a tailored Benders decomposition algorithm enhanced with acceleration techniques to solve the large-scale robust optimization problem.
- We conduct numerical experiments using real data from multiple cities and compare our response efficiency and algorithm performance with existing studies.

# 2 Model Development

In this section, we develop the optimization model for TUEMP. We describe the nature of our problem in Section 2.1. In Section 2.2, we present the robust optimization model for TUEMP.

## 2.1 Problem statement

Consider a complete network $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ with a physical node set $\mathcal{N}$ and a direct arc set $\mathcal{A} = \{(i,j)|\forall i,j \in \mathcal{N}, i \neq j\}$. A node $i \in \mathcal{N}$ can be visited by trucks and UAVs and an arc $(i,j) \in \mathcal{A}$ is a direct link between two nodes. In disasters, certain nodes will experience emergencies and propose demands for humanitarian relief assistance, resulting in a set of demand nodes $\widetilde{\mathcal{N}}$. To serve all demand nodes timely, the Emergency Management Department (EMD) conducts pre-disaster relief deployments at designated depots and makes post-disaster assignments using a fleet of trucks $\mathcal{K}$ and UAVs $\mathcal{U}$. We apply a flexible truck-UAV collaboration strategy, where UAVs can launch and land on different nodes and trucks.

We aim to minimize total emergency management cost by optimizing four decisions: (i) a relief location plan $\mathbf{y}$, which opens a set of depots to store relief with depot operation cost $\pi^y(\mathbf{y})$; (ii) a truck routing plan $\mathbf{x}$, which decides the travel routes of trucks starting from their depots with truck travel cost $\pi^x(\mathbf{x})$; (iii) a task assignment plan $\mathbf{z}$, which assigns delivery tasks to trucks and UAVs with fleet operation cost $\pi^z(\mathbf{z})$; and (iv) a service schedule $\mathbf{a}$, which determines the time to serve each demand node, with a delay penalty $\pi^a(\mathbf{a})$ on the total service delay multiplied by a high value of time $\beta$.

Uncertainties surrounding demand and travel time present challenges for model development. Unpredictable disaster scenarios lead to uncertain demand requests, including the relief quantity and the due time. Additionally, disasters affect travel time for the fleet due to damaged roads and adverse weather conditions. Consequently, post-disaster decisions and costs are not deterministic, but depend on the uncertain information. To address these complexities, we develop a robust optimization framework focusing on worst-case costs.

## 2.2 A robust optimization model for TUEMP

In this subsection, we develop a robust optimization model for the TUEMP. We begin by introducing the scenario-based characterization of the uncertainties. Let random vector $\boldsymbol{\xi}$ denote all the uncertain parameters on relief demand and travel time. We assume that the number of possible disaster scenarios is finite, which can be characterized by leveraging historical data. Let $\mathcal{E}$ denote the finite scenario set. Each scenario $e \in \mathcal{E}$ gives a realization of $\boldsymbol{\xi}$, denoted as $\boldsymbol{\xi}^e$. Consequently, the set of possible realizations of the random vector $\boldsymbol{\xi}$ is expressed by $\{\boldsymbol{\xi}^e|e \in \mathcal{E}\}$. Incorporating these uncertainties, we present our robust optimization model in (1)-(5).

$$\min_{\mathbf{y},\mathbf{x}} \pi^y(\mathbf{y}) + \pi^x(\mathbf{x}) + \max_{\boldsymbol{\xi} \in \{\boldsymbol{\xi}^e|e\in\mathcal{E}\}} \min_{\mathbf{z},\mathbf{a}} \widetilde{\pi}^z(\mathbf{z}) + \widetilde{\pi}^a(\mathbf{a}) \tag{1}$$

$$
\begin{aligned}
& s.t. \\
& \quad \text{Constraints for relief location plan } \mathbf{y} && (2)\\
& \quad \text{Constraints for truck routing plan } \mathbf{x} && (3)\\
& \quad \text{Constraints for task assignment plan } \mathbf{z} && (4)\\
& \quad \text{Constraints for service schedule } \mathbf{a} && (5)
\end{aligned}
$$

The objective (1) minimizes the worst-case total cost. Here, deterministic costs include relief location cost $\pi^y(\mathbf{y})$ and truck routing cost $\pi^x(\mathbf{x})$, while task assignment cost $\widetilde{\pi}^z(\mathbf{z})$ and schedule delay penalty $\widetilde{\pi}^a(\mathbf{a})$ depend on scenario realization $\boldsymbol{\xi}^e$. Tailored constraints are involved in (4) and (5) to impose UAV limitations and ensure the spacial-temporal synchronizations between trucks and UAVs.

However, the "min-max-min" structure of the objective function (1) makes the model intractable, underscoring the need for practical and efficient solution approaches.

# 3 Solution Approach

In this section, we develop a tailored Benders decomposition algorithm to solve the robust optimization model for TUEMP. In Section 3.1, we reformulate the model into two stages. Based on the reformulation, we illustrate the process of Benders decomposition in Section 3.2. In Section 3.3, we enhance the Benders-based algorithm with acceleration techniques.

## 3.1 Two-stage reformulation

In this subsection, we decompose model (1)-(5) into two stages: pre-disaster planning and post-disaster response.

The stage-one problem minimizes the worst-case total cost by deciding the deterministic relief location plan and truck routing plan, as is presented in (6)-(8). Here, the auxiliary variable $\Pi_2$ denotes the worst-case post-disaster cost and $\pi_2(\mathbf{y}, \mathbf{x}, \boldsymbol{\xi}^e)$ denotes the expected post-disaster cost given a stage-one solution $(\mathbf{y}, \mathbf{x})$ and a scenario realization $\boldsymbol{\xi}^e$.

$$
\begin{aligned}
& \min_{\mathbf{y},\mathbf{x}} \pi^y(\mathbf{y}) + \pi^x(\mathbf{x}) + \Pi_2 && (6)\\
& s.t. \\
& \quad (2), (3), && (7)\\
& \quad \Pi_2 \geq \pi_2(\mathbf{y}, \mathbf{x}, \boldsymbol{\xi}^e) \qquad \forall e \in \mathcal{E} && (8)
\end{aligned}
$$

Given a stage-one decision $(\mathbf{y}, \mathbf{x})$ and a scenario realization $\boldsymbol{\xi}^e$, the stage-two problem minimizes the post-disaster cost with a task assignment plan and service schedule, which is presented as:

$$
\pi_2(\mathbf{y}, \mathbf{x}, \boldsymbol{\xi}^e) = \min_{\mathbf{z},\mathbf{a}} \widetilde{\pi}^z(\mathbf{z}) + \widetilde{\pi}^a(\mathbf{a}) \tag{9}
$$

$$
s.t. \text{ (4) and (5).} \tag{10}
$$

## 3.2 Benders decomposition

The stage-one problem cannot be solved directly before the randomness is known, as constraints (8) rely on the solution of stage-two problems. To address this issue, we apply a Benders decomposition approach, which is an effective strategy for solving large-scale mixed-integer linear programs (MILPs) under uncertainty. This method decomposes the model into a relaxed master problem (RMP) and a series of subproblems (SPs), which are solved iteratively. Solving the RMP generates a candidate solution and establishes a lower bound. Subsequently, the SPs are solved using the candidate solution as parameters, yielding upper bounds and generating Benders optimality cuts. These cuts are then incorporated into the RMP, and the process iterates until convergence is achieved.

Following the two-stage reformulation, the SPs are stage-two problems (9)-(10) for each scenario. The RMP is the stage-one problem with constraints (8) substituted by Benders optimality cuts. Since both the RMP and SPs involve integers, we employ integer L-shaped cuts introduced by Laporte and Louveaux (1993). Each Benders optimality cut provides a lower bound for $\Pi_2$. By adding Benders optimality cuts into the RMP, the approximation of constraints (8) is gradually enhanced, and the algorithm approaches optimum.

Following Benders decomposition, both the RMP and SPs are deterministic, ensuring tractability. However, the RMP involves the NP-hard VRP and numerous Benders optimality cuts. Moreover, a substantial number $|\mathcal{E}|$ of SPs are solved in each iteration, which are MILPs. Consequently, the decomposed model is computationally challenging, especially in large-scale networks. These challenges necessitate tailored enhancements.

### 3.3 Tailored Benders-based algorithms

In this subsection, we enhance our Benders decomposition algorithm with tailored acceleration techniques.

*Lower bound lifting*. Due to the complete exclusion of stage-two information from the RMP, a large number of iterations and optimality cuts are needed to enhance the lower bound and narrow the optimality gap. To address this, we generate initial optimality cuts using a relaxation of the stage-two cost which is easy to compute. These cuts lift up the lower bound and reduce the iterations needed for convergence.

*Scenario sorting*. Traditionally, for any candidate solution, one Benders optimality cut is obtained from the worst-case scenario. Nevertheless, we sort the scenarios by severity, and prioritize generating cuts from potentially worse scenarios. The non-optimal solutions are cut off without solving all SPs, and the optimal solution is identified when no violated cut can be found from any scenario. This technique reduces the number of SPs to solve while ensuring optimality.

## 4 Numerical experiments

In this section, we evaluate the emergency management system using data from disaster-prone cities like Hong Kong, Shanghai, and the Gulf Coast, which includes roads, census data, meteorological conditions, geographic features, and historical disaster records. We conduct case studies with the data to demonstrate the superiority of our truck-UAV collaboration strategies and solution algorithms in comparisons with those of other studies. Evaluation indicators include timeliness, total cost, and computational efficiency.

Sensitivity analysis is conducted on the value of time $\beta$. A larger $\beta$ indicates that timeliness is more critical. Consequently, the EMD is willing to make larger investments to shorten the delay.

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