Machine learning to calibrate roads' delay functions using count data

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Abstract:

Roads' delay functions are indispensable for traffic models but their traditional calibration method (based on traffic survey) is data-intensive, time-consuming and costly. As a quick, efficient and effective alternative, we propose a method based on the readily available loop-detector traffic count data. In this method, the aim is to update the delay functions so that the traffic model generates similar data as that of traffic flow recorded at the loop-detectors. The proposed method is formulated as a Bilevel problem (NP-hard with highest computational complexity), and a hybrid solution algorithm based on machine-learning and optimization (ML-O) methods. The proposed methodology is tested using two case-studies, Sioux-Falls as a benchmark and Winnipeg as a real-life example, to demonstrate the practical merits of the method. The results show that for such an intractable problem, even at large scales (Winnipeg city), a very fine precision (say 1% error) is readily attainable.

Introduction: Traffic equilibrium models (also known as traffic assignment problems (TAP)) consists of two components to be developed and maintained, namely network supply and travel demand information, which require tedious, time-consuming, data-intensive and costly calibrations [1, 2]. In addition to road topography and configuration, the most important part of a network is the roads' delay functions which is also known as volume-delay functions (VDFs). These functions indicate a road's travel time (or delay) given the traffic flow (volume) passing through it. Calibration of the VDFs is generally based on an extensive traffic surveys to collect travel time and traffic counts for a limited number of roads as a sample representing the rest of the road network [2]. Traffic surveys can be done in a variety of ways such as plate-number recording, probe vehicles (for recording travel time), video detection and inductive loop detectors (for traffic count). For the latter, high installation and maintenance costs hinder the ubiquitous deployment of detectors all over the network [3, 4]. Thus, the coverage of such detectors is normally limited to a subset of links within a network [5, 6] or at actuated traffic signals controlled by traffic signal control technologies such as SCATS.

Methodology: Hence, the best bet is to calibrate the VDFs to the end-result of traffic assignment which is the traffic flow on the roads. In other words, to adjust the VDFs parameters to the extent that the outcome of the traffic assignment (roads' traffic flow) will be identical to the traffic count. Given this line of thought, this paper addresses the gaps in knowledge outlined above which are considered a solid motive to seek alternative and less-expensive calibration methods. By doing so, the information of the entire road network will be used for the VDF calibration (i.e. origin–destination OD) [1, 7, 8] which addresses the concern of losing spatial correlation as mentioned above.

u-VDF: $$\underset{(i,j)\in A, r=1..4, s=1..5}{Minimize} Z(x_{ij}, y_{ij}^{ar}, y_{ij}^{bs}) = \sum_{(i,j)\in A'} (x_{ij} - \bar{x}_{ij})^2, \quad (1)$$

subject to

$$a_{ij} = 2^3 . y_{ij}^{a4} + 2^2 . y_{ij}^{a3} + 2^1 . y_{ij}^{a2} + 2^0 . y_{ij}^{a1} \qquad (i,j) \in A'' \quad (2)$$

$$100. b_{ij} = 2^4 . y_{ij}^{b5} + 2^3 . y_{ij}^{b4} + 2^2 . y_{ij}^{b3} + 2^1 . y_{ij}^{b2} + 2^0 . y_{ij}^{b1} \qquad (i,j) \in A'' \quad (3)$$

$$a_{ij}^l \le a_{ij} \le a_{ij}^u, b_{ij}^l \le b_{ij} \le b_{ij}^u \qquad (i,j) \in A'' \quad (4)$$

$$y_{ij}^{ar} = \{0,1\}, y_{ij}^{bs} = \{0,1\} \qquad (i,j) \in A'', r = 1..4, s = 1..5 \quad (5)$$

$$\underset{x_{ij}}{Mimimize} \sum_{(i,j)\in A} \int_0^{x_{ij}} (a_{ij} + b_{ij} . x_{ij}^4) . dx \quad (6)$$

subject to

$$\sum_{p\in P_w} h_p = q_w \qquad w \in W \quad (7)$$

$$x_{ij} = \sum_{w\in W} \sum_{p\in P_w} h_p \delta_{ij,p} \qquad (i,j) \in A \quad (8)$$

$$x_{ij} \ge 0 \qquad (i,j) \in A \quad (9)$$

In the upper level, Equation (1) is to minimize the gap between target traffic volumes ($\bar{x}_{ij}, (i,j) \in A'$) and corresponding TAP-generated traffic volumes $x_{ij}, (i,j) \in A'$ as a squared function (or an error index). As discussed before, Constraints (2) and (3) link the real decision variables (a_{ij}, b_{ij}) up to the equivalent binary decision variables $y_{ij}^{ar}, y_{ij}^{bs}, r = 1..4, s = 1..5$. Constraint (4) enforces lower bound and upper bound to the VDF's parameters (a_{ij}, b_{ij}). Constraint (5) makes sure that the designated binary variables are in fact binary. In the lower level ((6) to (9)), the traffic assignment problem (TAP) is formulated based on the Beckmann's user equilibrium (UE) traffic flow [9]. The main idea behind the UE traffic flow is that drivers always choose the shortest paths.

ML-O solution algorithm: As discussed above, the u-VDF as a Bilevel problem is NP-hard [10]. Generally speaking, to cope with NP-hard computational complexity of a Bilevel problem, the consensus is to decompose the problem into two sub-problems corresponding to two upper and lower bound values of the objective functions. Such schemes usually become time-consuming for large-sized networks. As an alternative, the authors have developed a hybrid method for general Bilevel problems consisting of a supervised learning technique and an integer programming problem [11], denoted by ML-O, which is retrofitted in this study as a solution algorithm for the u-VDF problem. The ML-O algorithm initiates with a feasible binary solution $(y_{ij}^{ar}, y_{ij}^{bs})$ for which, the TAP is solved to calculate Z, the value of the objective function (1). These data (Z and $(y_{ij}^{ar}, y_{ij}^{bs})$) are then used to train a multivariate linear regression model as a function of the decision variables:

$$\bar{Z} = \sum_{(i,j)\in A''} b_{ij}^{ar} . y_{ij}^{ar} + \sum_{(i,j)\in A''} b_{ij}^{bs} . y_{ij}^{bs} \qquad (10)$$

where $\bar{Z}$ is a linear approximation or a surrogate function of the original objective function Z and b_{ij}^{ar}, b_{ij}^{bs}, are parameters to be calibrated. In other words, given a set of training data (Z and $(y_{ij}^{ar}, y_{ij}^{bs})$) an estimate of the parameters $(b_{ij}^{ar}, b_{ij}^{bs})$ is sought. The calibration process is equivalent to solve for a quadratic minimization of the gap between $Z$and $\bar{Z}$ subject to several linear constraints. This is a convex problem and hence easy to solve. The next step is to arrive at a new feasible and possibly a better binary solution for which we construct an integer linear programming (ILP) problem as follows. The ML-O postulates that the regression model, which is a linear function of decision variables, is an approximation of the original objective subject only to the binary constraints of the original problem.

Numerical tests: To demonstrate the feasibility of the approach, we test the proposed methodology in two cases, namely: a medium-sized network, Sioux-Falls and a large-sized network, Winnipeg. For the sake of brevity we only report on Winnipeg results. To arrive at a precise numerical result when solving a TAP, as suggested by [12], we set "relative gap" of the size of 0.0001 as a termination criterion. We employ a desktop computer with 64.0 GB RAM and CPU processor of Intel Xeon 3.70 GHz. The algorithm is coded with python linked to MS-Excel as an interface as well as MS-Access a database. The computer program is interfaced to EMME 3 to solve TAPs for the Winnipeg. (for the Sioux-Falls we use a GAMS code developed by Ferris, et al. [13]) . The code also calls on GAMS (Baron solver [14]) to solve the u-VDF-ILP for solving the multivariate regression.

The Winnipeg network consists of 943 nodes and 3,095 directional link roads. We consider a scenario in which traffic authorities seek updating VDF of a number of selected roads in the Central Business District (CBD) given traffic count data of a limited number of loop detectors (traffic count posts). Figure 1 exhibits the location of traffic count posts (40 posts) as well as 106 main roads in the CBD whose VDF parameters will be re-calibrated. Note that, we excluded the empty or less congested roads of the CBD as their travel times are largely governed by their respective traffic volumes. Even by looking at the BPR function (i.e., $a_{ij} + b_{ij}.x_{ij}^4$), it is clear that when the traffic volume is low ($x_{ij}^4 = 0$) , it is the first term of the function (a_{ij}) (the free-flow travel time) which governs the travel time equation. As can be seen, a_{ij} (or free-flow travel time) represents the physical characteristic of the respective road and not the congestion level (note that 106 roads are the result of filtering the CBD roads over volume-per-capacity ratio of 80% which was done to exclude less-congested roads).

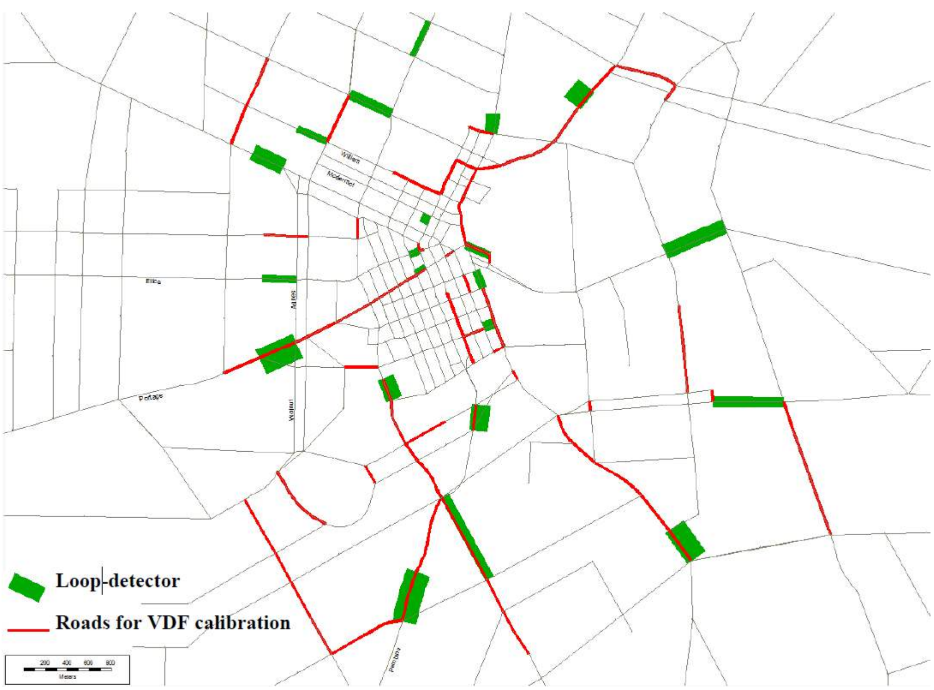

Figure 1 Winnipeg case study, location of 40 loop-detectors and 106 roads to VDF-recalibration

We leave the first term of the VDF intact and try to re-calibrate or update the function on their second term (i.e. b_{ij}). We initialize VDFs of the CBD's main roads (106 links) on their $b_{ij}$and set the target traffic volumes of the count posts (40 posts) to be those of the traffic volume of the original scenario. In the original scenario, the b_{ij} varies from 0.15 to 0.63, hence it suffices to represent it with 6 additional binary decision variables (in other words, we are testing the algorithm to see if it can reproduce the very same original traffic flow of the 40 roads). As a result, the total number of decision variables will be 636 (=6×106), based on which we run the ML-O algorithm for 6500 iterations. Figure 2 illustrates the convergence of the value of the objective function which exhibits a quick convergence after around iteration 1,200th. The best solution yields a gap error of 3,655 equivalent to average 1.09% error across the traffic count posts (see Figure 3), which is a small error and an acceptable result. As can be seen from Figure 3, the error or differences between model generated volume and count volume on the individual road is also negligible which is highly assuring

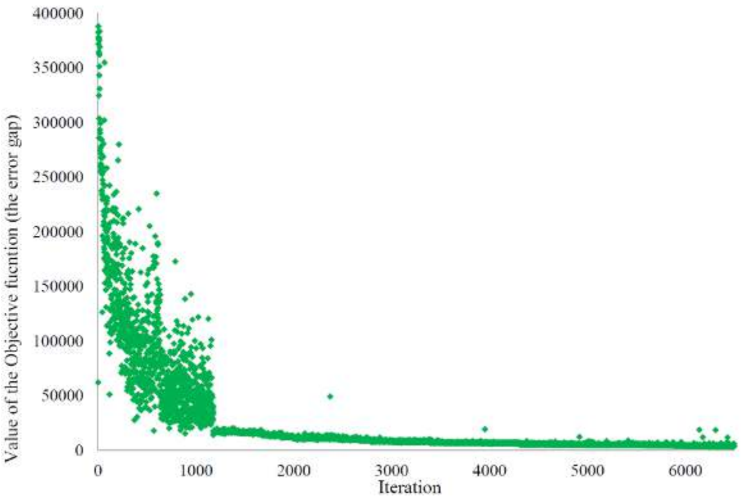

Figure 2 Winnipeg case-study, convergence of the value of the objective function

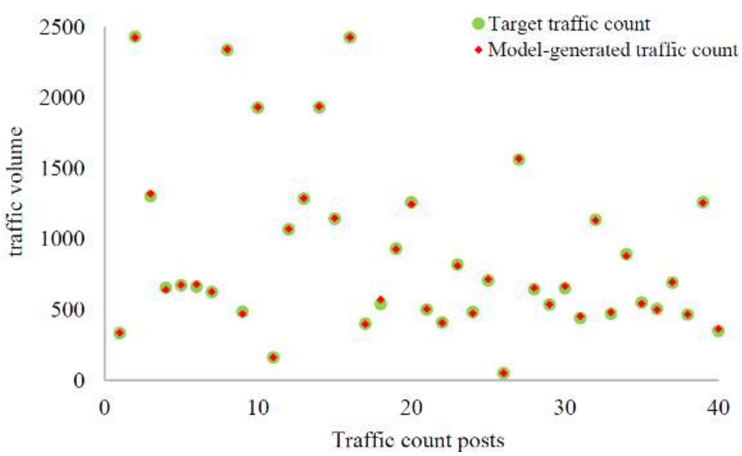

Figure 1 Winnipeg Case-study, model-generated traffic volume versus target volumes

6. Conclusion

The aim is to update a traffic assignment model (i.e. updating the VDFs) to regenerate traffic flow close to the target traffic count data. Hence, updating the VDFs (u-VDF problem) was formulated as a Bilevel problem to minimize a gap (error) function in the upper level while accounting for the traffic assignment problem (TAP) as a subproblem in the lower level. The u-VDF is a NP-hard problem (a term to highlight highest computational complexities), in that sense that respective problem for large-sized examples cannot be solved efficiently using exact methods. Therefore, heuristic methods are recommended. As such, we employed a hybrid algorithm based on machine-learning principles and traditional optimization methods (ML-O). The ML-O is an iterative procedure deciphering the NP-hard problem into two tractable problems to be solved alternately. The kernel of the ML-O is to learn from the wealth of data collected from previous iterations being stacked up as the algorithm proceeds. The proposed methodology was then tested using two case-studies, Sioux-Falls as a benchmark dataset and Winnipeg, a real dataset. The results corroborate the convergence of the algorithm to find some very good solutions (i.e. close to the global optimal) at affordable computational time.

The main contributions of the paper can be summarized as follows:

- The proposed methodology is tailored to real life problem.

- It is a modularized algorithm, in the sense that the TAP can be replaced by any other traffic simulation model (say a meso or micro model) which enhances flexibility, compatibility and applicability of the methodology.
- The proposed algorithm is also amenable to parallel computations (i.e. several modules can be executed simultaneously in parallel) as a mean to reduce computational time.

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